## Strategy #2 for Solving Tough Problems: If the General Problem Seems too Difficult, Consider Simpler but Related Problems

### *Problem (p. 40 #16)*

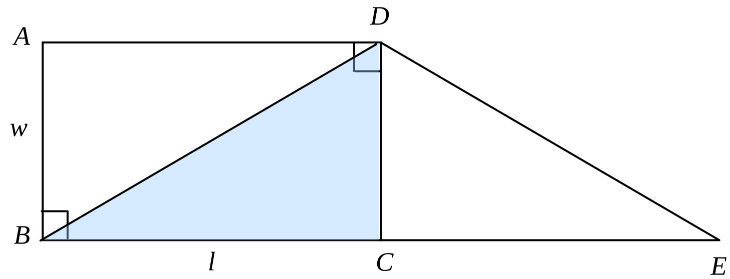
For a quadrilateral *ABCD*, describe a procedure for constructing a triangle whose area equals that of *ABCD*.

### *Analysis*

Initial attempts to solve this problem can lead to a great deal of frustration because we are not accustomed to working with general quadrilaterals. A very powerful strategy for dealing with this is to ***consider simpler but related problems***. This strategy is so powerful that it was even employed by Albert Einstein in his General Theory of Relativity.[1] Since we tend to feel more comfortable working with special quadrilaterals such as rectangles, parallelograms and trapezoids it makes sense to investigate these special cases first!

### *Simpler but Related Problem # 1*

Construct a triangle whose area is equal to that of rectangle *ABCD*.




#### *Solution*

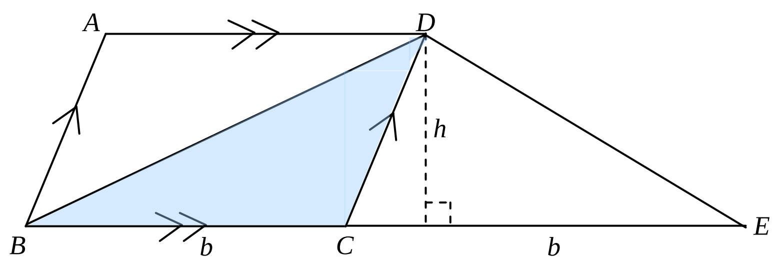
Construct a rectangle ABCD with length *l* and width *w*. Then extend line segment *BC* to *E* in such a way that *CE* = *BC*. Therefore,

$$ABCD = lw$$
and
$$\Delta DBE = ½(2l)w = lw,$$

which means that *ABCD* = Δ*DBE*. //

### *Simpler but Related Problem # 2*

Construct a triangle whose area is equal to that of parallelogram *ABCD*.




#### *Solution*

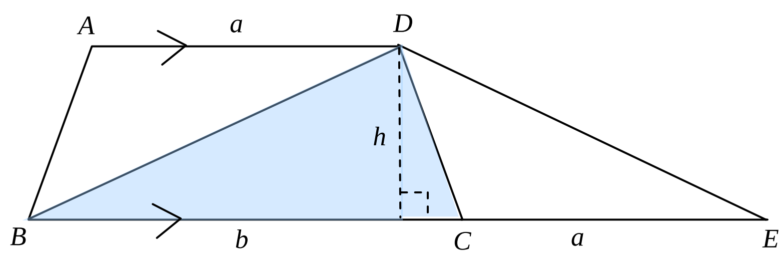
Construct parallelogram *ABCD* with base length *b* and height *h*. Then extend line segment *BC* to *E* in such a way that *CE* = *BC*. Therefore,

$$ABCD = bh$$
and
$$\Delta DBE = ½(2b)h = bh,$$

which means that *ABCD* = Δ*DBE*. //

### *Simpler but Related Problem # 3*

Construct a triangle whose area is equal to that of trapezoid *ABCD*.




#### *Solution*

Construct trapezoid *ABCD* with parallel sides of length *a* and *b* and height *h*. Then extend line segment *BC* to *E* in such a way that *CE* = *AD* = *a*. Therefore,

$$ABCD = ½(a+b)h$$
and
$$\Delta DBE = ½(a+b)h,$$

which means that *ABCD* = Δ*DBE*. //

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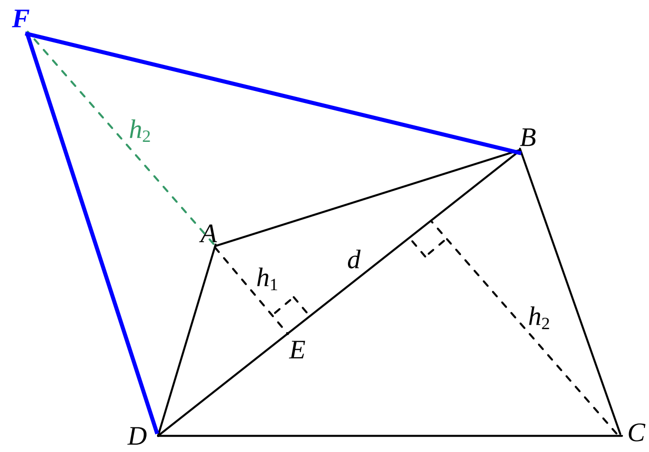
[1] Einstein's third postulate of the General Theory of Relativity expresses the equivalence of constant acceleration and uniform gravitational fields. This allowed him to transform the analysis of objects in motion in a uniform gravitational field, a very difficult undertaking, to the study of uniformly accelerated objects, a much more straightforward and better understood endeavour.

## *Back to the Real Problem (p. 40 #16)*

For a quadrilateral *ABCD*, describe a procedure for constructing a triangle whose area equals that of *ABCD*.




### *Solution*

After solving the special cases shown on the previous page, this problem becomes much more manageable. The insight gained from the special cases helps us to see this situation much more clearly.

Construct general quadrilateral *ABCD* and diagonal *BD*. This produces two triangles $\Delta ABD$ and $\Delta CBD$, both with base *BD*. Let $d$ represent the length of *BD* and let $h_1$ and $h_2$ represent the heights of $\Delta ABD$ and $\Delta CBD$ respectively.

Now extend *EA* to *F* in such a way that the length of *EF* is $h_1 + h_2$. Then,

$$\begin{aligned} ABCD &= \Delta ABD + \Delta CBD \\ &= \tfrac{1}{2}dh_1 + \tfrac{1}{2}dh_2 \\ &= \tfrac{1}{2}d(h_1 + h_2) \end{aligned}$$

Also,

$$\Delta FDB = \tfrac{1}{2}d(h_1 + h_2)$$

Therefore, $ABCD = \Delta FDB$. //