

MATHEMATICAL MISCONDUCT

Introduction

Although creativity is an essential part of the problem solving process, students need to keep in mind that mathematical statements *can never be assumed to be true!* Before we can apply any rule in mathematics, we first need to verify that the “rule” is correct! Unfortunately, many students *blindly invent* and *carelessly apply* “rules” that they have *never bothered to confirm*. This usually leads to ridiculous results!

1. To help you gain an appreciation of how important this is, complete the following table. Indicate whether each of the following mathematical statements is true or false. Prove the true statements and provide counterexamples or explanations for the false statements. (Many of these examples are taken from the *Mathematics Teacher*, January 1993.)

<i>Mathematical Statement</i>	<i>True or False?</i>	<i>Proof, Counterexample or Explanation</i>
(a) $\frac{3}{a} + \frac{3}{b} = \frac{3}{a+b}$		
(b) $\frac{a}{c} + \frac{b}{d} = \frac{a+b}{c+d}$		
(c) $\sqrt{a^2 + b^2} = a + b$		
(d) $\sqrt{a^2 - b^2} = a - b$		
(e) $(a+b)^2 = a^2 + b^2$		
(f) $(a-b)^2 = a^2 - b^2$		
(g) $\frac{a^2}{b^2} = \frac{a}{b}$		
(h) $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$		
(i) $\frac{a+\cancel{b}}{\cancel{b}} = a$		
(j) $\frac{1}{a+b} + (a+b)^{\cancel{3}} = a+b$		

<i>Mathematical Statement</i>	<i>True or False?</i>	<i>Proof, Counterexample or Explanation</i>
(k) $\sqrt{a}(a) = a^2$		
(l) $\sqrt{a}(a) = a$		
(m) $\frac{1}{3}(-6)^3 = -2^3$		
(n) $a^{\frac{2}{3}} = \frac{a^2}{a^3}$		
(o) $\frac{\sin a}{\sin b} = \frac{a}{b}$		
(p) $\frac{\sin \cancel{a}}{\cancel{a}} = \sin$		
(q) $\frac{\sin \cancel{a}}{\cancel{a}} = \sin 1$		
(r) $\frac{\sin 2\cancel{a}}{\cancel{a}} = \sin 2$		
(s) $\sin 2a = 2 \sin a$		
(t) $\sin(a+b) = \sin a + \sin b$		
(u) $\sin^2 a = \sin a^2$		
(v) $\frac{\sin \cancel{\theta}}{\cos \cancel{\theta}} = \tan$		
(w) $(\sin a)(\sin a) = \sin a^2$		

<i>Mathematical Statement</i>	<i>True or False?</i>	<i>Proof, Counterexample or Explanation</i>
(x) If $a + b = 0$, then $a = 0$ or $b = 0$.		
(y) If $x(x - 2) = 24$, then $x = 24$ or $x - 2 = 24$.		
(z) $abc = (ab)(ac)$		
(aa) If $\log a = b$, then $a = \frac{b}{\log}$.		
(bb) $\frac{t + u}{t + v} = \frac{t}{v}$		
(cc) $a^{-2} = -a^2$		
(dd) $(\sin a)(\sin 2a) = 2 \sin^2 a$		
(ee) $2a^{-1} = \frac{-1}{2a}$		

2. Correct all the false statements in question 1.